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Registration of Images To Lidar and GIS Data Without Establishing Explicit Correspondences

Gabor Barsai, Alper Yilmaz, Panu Srestasathiern, and Sudhager Nagarajan

Abstract
Recovering the camera orientation is a fundamental problem in photogrammetry for precision 3D recovery, orthophoto generation, and image registration. In this paper, we achieve this goal by fusing the image information with information extracted from different modalities, including lidar and GIS. In contrast to other approaches, which require feature correspondences, our approach exploits edges across the modalities without the necessity to explicitly establish correspondences. In the proposed approach, extracted edges from different modalities are not required to have analytical forms. This flexibility is achieved by minimizing a new cost function using a Bayesian approach, which takes the Euclidean distances between the projected edges extracted from the other data source and the edges extracted from the reference image as its random variable. The proposed formulation minimizes the overall distances between the sets of edges iteratively, such that the end product results in the correct camera parameters for the reference image as well as matching features across the modalities. The initial solution can be obtained from GPS/IMU data. The formulation is shown to successfully handle noise and missing observations in edges. Point matching methods may fail for oblique images, especially high oblique images. We eliminate the requirement for exact point-to-point matching. The feasibility of the method is experimented with nadir and oblique images.

Introduction
Registration is the process of aligning two or more datasets acquired for the same site with different coordinate systems to a single coordinate system. Considering the ever increasing amount of multi-modal datasets with different geometric, radiometric, temporal, and thematic resolutions, it is important to register these datasets to a single coordinate system by registration (Sester et al., 1998), which in turn provides us with the ability to exploit the advantages offered by each of the different modalities (Schenk and Csathó, 2002). The registration between such datasets is traditionally divided into four steps, namely: feature extraction, matching, transformation, and resampling (Zitová and Flusser, 2003). In this paper, we discuss a nontraditional method for registering aerial images to GIS (Geographical Information Systems) and lidar (Light Detection And Ranging) data. This is a continuation of the paper on view invariant shape recognition (Yilmaz and Barsai, 2008). This previous paper studied freeform matching using Fourier descriptors.

Registration of aerial images is an important task in photogrammetry, as it is a pre-requisite for 3D surface recovery and orthophoto generation, as well as performing higher level inference tasks such as object recognition. An image is traditionally registered to a reference system using a set of Ground Control Points (GCPs) and corresponding pixels in the image. With the advancement in GPS/IMU (Global Positioning System/Inertial Measurement Unit) technology, exterior orientation parameters (EOPs) are obtained from the instrument measurements and aerial images are registered on the fly, also called direct orientation. A major drawback of direct orientation is the computation of EOP without considering the interior orientation parameters (IOPs) that continually changes due to environmental and mechanical factors (Schenk, 1999). However, the EOP derived from direct orientation can be used as initial approximations for precise mapping applications.

For indirect orientation, it is possible to use information extracted from other datasets such as lidar and GIS. While very accurate GIS and lidar datasets are publicly available from USGS (United States Geological Survey) and state agencies for any given area, they are usually not used for registration due to the fact that establishing correspondences between an image and GIS/lidar is a challenging problem and is an ongoing research (Heipke, 1997) (Shan and Toth, 2009) (Sourimant et al., 2011) (Bartie et al., 2011). In recent years, researchers have exploited the features extracted from the lidar data, such as lines and planes, to register them with the images (Habib et al., 2007) (Habib et al., 2005) (Jaw and Wu, 2006) (Schenk and Csathó, 2002) (Nagarajan and Schenk, 2016). Extracting linear and planar features from lidar data, however, is not always intuitive due to surface patterns, noise, and lidar point density. In contrast to registration of lidar with images, the registration of GIS data with images has received less attention except only a few attempts such as (Sester et al., 1998, Shan, 2000) and (Chavathe, 2007). GIS features and the features extracted from a different sensory data may not be the same due to different sampling of the real world (Schenk and Csathó, 2007). The number of vertices that constitute a shape can also make features to look different. This paper demonstrates a novel method to register an image with the GIS and lidar data without establishing correspondences by eliminating the feature matching step. Image registration methods can broadly be categorized based on (a) the dataset representation, (b) the model for establishing correspondences and (c) the choice of mathematical models (Nagarajan, 2010). In the case when...
registration is performed between different modalities, it is important to relate information extracted from the complimentary domains where one modality provides information that is not available in the other one. Due to the overwhelming volume of research that has been performed on registration, this paper discusses only the previous research that is directly relevant to the methodology described in the remainder of the text. The interested reader can find overviews of registration from a variety of perspectives in (Campbell and Flynn, 2001; Huber and Hebert, 2003) and (Li et al., 2008). Here, we will concentrate on a feature-based approach that matches free-form edges without the need for explicit, feature to feature correspondences across different modalities.

Most photogrammetric applications depend on the use of distinct points (Habib et al., 2002), which can be extracted manually, or automatically using Moravec point operator (Moravec, 1977), Förstner point operator (Förstner and Gülch, 1987), Harris corner detector (Harris and Stephens, 1988) or Scale Invariant Feature Transform (SIFT) (Lowe, 2004). While point based methods, which depend on features extracted by “experienced personnel”, produce high precision results in a traditional setting, their accuracy, however, drops significantly when the process is automated. This performance degradation is due to challenges stemming from several factors including: partial occlusions, viewpoint changes, varying illumination, cluttered backgrounds, and complimentary information acquired from different sensor modalities (Remondino et al., 2008). In order to develop more accurate techniques, researchers have more recently resorted from points to higher level features, such as lines (Habib et al., 2003) (Schenk, 2004), surfaces (Jaw and Wu, 2006), curves (Hartley and Zisserman, 2000), manholes (Drewniok and Rohr, 1997) and road junctions (Pedersen, 1996). These features provide higher level of redundancy and decrease the effect of errors associated with noisy point locations and imperfect matches (Wang et al., 2008). All these methods and other similar approaches typically require user interaction to create local geometric models (Lübe and Ellenbeck, 1996) (Doucette et al., 2008), and to find and match elongated features, such as roads (Habib et al., 2003). Automated photogrammetric methods are based on individual point matching, but point matching fails for oblique images (Tuytelaars and Mikolajczyk, 2008) due to higher distortions. In addition, GPS-IMU accuracy is dependent on the instrument itself, as shown in (Leberl et al., 2010). Add to that the IOPs may change during the use of the camera: air pressure change, shaking of the camera could change over the time it takes to obtain the images, making individual point matching very dependent on the quality of the individual points.

For point-based methods applied on data from different modalities, the ICP (Iterative Closest Point) algorithm (Besl and McKay, 1992) is considered to be one of the most popular methods for many years with applications in registering targets to lidar scans (Barnea and Filin, 2008) (Wang and Brenner, 2008). The ICP algorithm finds the best correspondences between two point sets by iteratively determining the translation and rotation parameters of a 2D or a 3D rigid body transformation by minimizing the distance between the two data sets. The ICP algorithm, however, assumes that one point set is a subset of the other set, which is not the case when, for instance, an image is registered to 3D points from a lidar dataset. In addition, photogrammetric points may even not have any conjugate points among the lidar points. Despite its common application, ICP is used when the transformation between two sets contains rotation and translation, and has not been shown to provide accurate results when other transformations, such as scaling, is observed (Nagarajan, 2010). Due to this observation, there has been a quantifiable number of papers on registering two lidar datasets to each other using ICP (Habib et al., 2010) (Glennie et al., 2014), while papers in its use for across-modality registration are limited (Toth et al., 2008).

In this paper, we propose a new approach to estimate the EOP and IOP by registering images to lidar or GIS datasets. In the following discussion, we refer to EOP and IOP as the camera parameters. The proposed work shares the underlying idea of ICP and minimizes Euclidean distances between feature sets; however, there are several key differences. In our proposed approach, we do not assume point correspondences, instead we generate a distance map and revert from a correspondence model to a Bayesian inference model, which measures the likelihood of information from the two modalities in terms of Gaussian distributions using the distance map as its variable. Due to the proposed treatment of the problem, any type of features including, points, lines, and free-form lines can be used in the registration process. This added flexibility can provide additional redundancy which will be experimentally shown to improve the precision in estimation of the camera parameters. The proposed method will demonstrate its insensitivity to noise and incomplete data. The proposed method is similar to (Lowe, 1991), with a different application field and different data. (Lowe, 1991) also attempts to match edges and use the Levenberg-Marquardt least squares minimization method for computer graphics purposes for model fitting, not for orientation parameter determination, although a solution to the orientation parameters is a side-product of the method.

The following discussion is organized as follows. In the next section, we introduce the data modalities and the preprocessing steps applied on these data prior to registration. Followed by details of the collinearity model, the distance transform, and the Bayesian modeling of the registration process as they are used in the proposed scheme. An experimental demonstration of the approach is given in the next Section comparing the proposed method with the manual registration which serves as the baseline technique. Finally, we provide concludes.

### Sensor Modalities and Preprocessing

In order to register images, we consider two possible data modalities including lidar and GIS. Figure 1 shows data for the California study site. Lidar, which is also referred to as LADAR or laser scanning, serves as an essential complement to photogrammetry for mapping and analyzing surfaces. The lidar technology provides an independent surface topography by means of laser ranging; hence provides a direct way to model 3D surface structure. This property has led to its wide popularity for forest and urban mapping. Registering lidar with images, however, has always been a challenge, which can be blamed on the difficulty in matching features between image and lidar. In contrast to the grid structure in images, experimental evidence supports that lidar point cloud is distributed sparsely over surfaces and the resulting 3D points generally do not correspond to feature points in images, (Leberl et al., 2010) investigates the differences and data collection rate between lidar and image data, concluding that image based data has a higher density and states on page 1128: “In summary the experimental evidence suggest that data from a fully digital photogrammetric workflow offer more detail at a comparable geometric accuracy than one can get from an aerial lidar-system.”

Given multimodal datasets of a scene, based on the sensors and filters and the knowledge of the human operator, the extracted surface features may look different, however, these are all different representations of the same real world surface (Schenk and Csatho, 2007). Following this conjecture, we consider the discontinuities in both the lidar and image. The
discontinuities in lidar data represent elevation differences of the surface, such as between the facades of a building. Depending on the viewpoint, a subset of these 3D discontinuities manifest themselves in the images as edges observed due to parallax. In order to detect discontinuities from the lidar data, we use the edges detected by thresholding the lidar elevation data. Figure 2 shows the result of edge detection on the image, the edges in the image and the projected lidar data correspond to the depth discontinuities. We should note that edges extracted from an image contain such edges, and include additional edges observed due to texture changes.

Similar to the segment boundaries of lidar elevation data, GIS contains vector data denoting the edges of surface features manually extracted by an operator. GIS data is usually orthographic in the form of a map and does not contain side views of buildings, hills or other tall objects. As a common practice, GIS may not contain elevation information; hence, cannot be directly projected to the image space. There are two possible ways to offset this problem so that the GIS can be registered to the ground. One possibility is to use the lidar data to define the GIS elevation. Good GIS data should register very closely with the lidar data as illustrated in Figure 3a and 3b, showing the two datasets against each other. This allows us the ability to select the elevation from the closest lidar point residing inside the polygon/polyline data. Alternatively, a generic elevation measured from spot-elevations on the ground can also be assigned to the GIS, such as from the elevation available from USGS maps from contours or spot points. With available elevation data, the GIS data provides adequate information, which can be fused with images for precise camera parameters estimation.

The edges in the image data, which include the edges from the depth discontinuities, are extracted using both the Canny and the mean-shift segmentation based edge detectors. These edge detectors provide results in different scales, which, as discussed in the next section, provides us with the ability to perform a coarse-to-fine parameter estimate. As seen in
Figure 4c, the mean shift based edge detector provides sparse but closed edge contours with less noise; hence it is suitable for estimating an initial approximation of the camera parameters. The drawback of the sparse edges, however, is its lack of detail. Once the initialization is performed, we use edges extracted from the Canny edge detector for fine estimation (see Figure 4b). For the edges detected by Canny, we perform edge linking to generate longer edge segments. It should be noted that the edges extracted from the image generally contain more edge segments than both the GIS and lidar edges. These additional edges may be observed due to texture, trees, and vehicles and can be filtered based on their length. In this paper, the length based filtering is performed by searching straight lines using the Hough transform (Hough, 1962). In particular, we use the camera parameters to determine the threshold for removing short edge segments (Ok et al., 2010). Ok et al. (2010) relies on the epipolar geometry and the idea of finding the overlaps of lines in different views and propose a normalization scheme to deal with the problems of the geometrical reliability of the line segments extracted from different views. We conjecture that longer edges correspond to buildings or long sidewalk edges.

After edges are pruned, the camera parameters are adjusted by selecting areas that are at least 30 m × 30 m, or about 7,500 pixels. These values represent an average size building on the study area, not cars or noise. An illustration of this process is shown in Figure 2b and 2c, where the threshold is set at 50 pixels. For the Ohio State study site, the images are captured at a flying height over 2,200 meters and have very small scale; we used a 50 meter threshold to remove outlier edges.

After the postprocessing, the resulting edge segments along with the edges extracted from the lidar or GIS modalities serve as the data to estimate precise camera parameters. Considering missing and incomplete edge segments across modalities, establishing correspondences is a very challenging task. In the following discussion, we introduce the proposed approach that eliminates the requirement to establish correspondences between these features, yet provides precise camera parameters.

Methodology

Once the edge segments are extracted from three different modalities, we conjecture that there exists an unknown conjugacy between subsets of edges across the modalities, such that some edges are considered noise or extra information that are not contained in all domains. Considering the paper is on estimation of camera parameters of images, we frame our discussion around using image as the first modality and using either

![Figure 3](image-url)  
**Figure 3.** Ideal case for GIS and lidar data that registers well. Black outline represent the GIS data, dots represent lidar data with different shades representing different elevations. A select region from (a) OSU study area, and (b) California San Bernardino study area.

![Figure 4](image-url)  
**Figure 4.** (a) Cropped image from the California study site image 425, (b) Edges detected using the Canny edge detector (Notice that some edges are not complete), and (c) Closed edges detected using mean-shift segmentation.
GIS or lidar as the second modality. In this setting, the edges extracted from the image are referred to as the image data, and the edges from either lidar or GIS are referred to as the ground data. The image and ground data pair initially is not registered, such that projecting the ground data to the image using the initial camera parameters results in misaligned edge pixels, as shown in Figure 5. In the following text, we first introduce how misaligned edge pixels are treated which is followed by a discussion on how the proposed mathematical model is defined and used to adjust initial camera parameters.

**Projection of the Ground Data**

Let there be two sets of edge points \( x_i \in \mathbb{R}^2 \) and \( X_i \in \mathbb{R}^3 \) which respectively belong to the image data and ground data. The 3D edge points \( X_i \) are assumed absolute and are adjusted, and their projections to the image will only conform to the image edges upon successful adjustment of the camera parameters. The projections from ground data to image domain is performed by applying the collinearity equations (Wolf and Dewitt, 2000):

\[
\begin{align*}
x'_i &= x_0 - f \left( \frac{(X_i - X_0) r_1 + (Y_i - Y_0) r_2 + (Z_i - Z_0) r_3}{(X_i - X_0) r_1 + (Y_i - Y_0) r_2 + (Z_i - Z_0) r_3} \right) \\
y'_i &= y_0 - f \left( \frac{(X_i - X_0) r_1 + (Y_i - Y_0) r_2 + (Z_i - Z_0) r_3}{(X_i - X_0) r_1 + (Y_i - Y_0) r_2 + (Z_i - Z_0) r_3} \right)
\end{align*}
\]

where \( X_i = (X_i, Y_i, Z_i) \) are ground points, \( (X_0, Y_0, Z_0) \) is the perspective center, \((x_0, y_0)\) is the principal point coordinates (PPC), \( f \) is focal length, and \( r_1 \) is the component at the \( i^{th} \) row and \( j^{th} \) column of the rotation matrix taking orientation angles \( \omega, \varphi, \kappa \) as its parameters. The equations given in Equations 1 and 2 are based on the principle that the perspective center, image point, and corresponding 3D point are collinear and provides a relation between the camera parameters, image and the 3D points. Hence, the parameters of the model include \( \mathbf{p} = [\omega, \varphi, \kappa, \ Xc, Yc, Zc, f, x_0, y_0] \). Note that we consider the image distortion and refraction in this model are removed, hence they are not part of the collinearity equation.

In a traditional photogrammetric solution, the 3D points, referred to as GCPs (Ground Control Points), and their conjugate image point coordinates are known and are used to adjust the parameters vector \( \mathbf{p} \). In this paper, we treat the ground data corresponding to lidar or GIS as a set of 3D points which serve a similar purpose as GCPs. The height of the lidar points are used to distinguish rooftops from ground. These points, however, do not have conjugate image points and establishing such conjugacies is nontrivial. Nonetheless, we project these 3D points to the image space and compute the distance from the closest image points lying on extracted image edges. The adjustment of the parameter vector \( \mathbf{p} \) provides a means to minimize the distance between the projected and image points. In the following discussion, we present an intuitive yet powerful technique to facilitate fast computation of the distances between the projected and image edge points without performing a brute-force search.

**Distance Transform**

Since conjugacies are not known and are hard to explicitly establish, we instead consider the distances from the projected edges to the image edges. The distance of a projected ground edge pixel, \( x'_i \) to the image edge pixels \( x_j \) can be computed using the Euclidean distance;

\[
d(x'_i ,x_j) = \min (x'_i-x_j)^2 + (y'_i-y_j)^2
\]

Estimation of this distance and selecting the closest point, when performed for all projected edge points individually, becomes similar to ICP estimation and is not feasible for a large set of points. Alternatively, we apply the distance transform to the image edges, which serves as a look up table for projected ground edges to estimate closest distances at no computational cost. This process, however, does not explicitly match the ground edges to image edges.

The distance transform is an operator applied to binary images (Rosenfeld and Pfaltz, 1968) and has been considered for computing the object skeleton (Sáude et al., 2006), enhancing images (Zeng and Hirata, 2003) and estimating shortest path between two image points (Kozinska et al., 1997). Let the image edges be set to 1 and non-edge pixels are set to 0 forming the binary image, such as the pixels lying on the dotted line in Figure 6a are set to 1 and remaining pixels are set to 0. The distance transform applied to this binary image generates another image, \( D \), which encodes the distances of all 0-valued pixels from the 1-valued pixels. The distances can be considered like intensity values, hence generates a distance map as shown in Figure 6a. An illustration of the distance for a 5 × 5 neighborhood is illustrated in Figure 6b. The distances,

![Figure 5. Misalignment problems of the ground data: (a) lidar, and (b) GIS, with the image data using the Initial GPS/IMU values. This figure includes sparse points that are easy to follow.](image-url)
The distance transform can be computed using various distance functions including but not limited to the Euclidean distance, chessboard distance, Chebyshev distance, Manhattan distance, Hausdorff distance (Meier, 1998), geodesic distance or the chamfer distance (Chetverikov and Khenokh, 1999). For details on different distance metrics and distance transform algorithms, we direct the reader to a comprehensive survey in (Cuisenaire, 1999).

In the next section, due to its use in the estimation of the camera parameters, we employ the Euclidean distance metric computing the distance transform and generate the transform using a fast algorithm described in (Maurer et al., 2003). In our implementation, we apply the distance transform only to the image edges, and not on the projected edges extracted from the ground data. This is due to the fact that the estimation of the camera parameters iteratively change the locations of the projected edges, while the pixel locations of the image edges remain constant. As a result the distance transform is only applied once and is not repeated at each iteration, hence reduces the computational time.

Mathematical Model

There are two general methods of parameter estimation. They are least-squares estimation (LSE) and maximum likelihood estimation (MLE). MLE is of fundamental importance in statistics, unlike LSE, which is primarily a descriptive tool (Myung, 2003). In MLE we seek the parameter values that are most likely to have produced the data. In LSE, on the other hand, we seek the parameter values that provide the most accurate description of the data, measured in terms of how closely the model fits the data under the square-loss function. We give a description of each setup below and show the results of the LSE, generally used in adjustment.

Direct conjugacies between the image data and the ground data are not known; hence, we treat the distances computed between the projections of the ground data and the image data as hypotheses. Note that the hypothesized distance for a projected ground datum also implies hypothesized corresponding image edge pixel. Since the initial camera parameters are approximate solutions, such implied correspondences, \( < x_j, x'i > \), become random variables whose likelihood can be computed assuming a zero mean normal distribution \( N(0, \sigma) \):

\[
p(< x_j, x'i >) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{d(x_j, x'i)^2}{2\sigma^2} \right)
\]

where \( d(.) \) is as given in Equation 3 and \( \sigma \) is the standard deviation of the distribution, which represents the uncertainty of observations and is set to a constant during the adjustment process. In the ideal case, when the ground and image data are precisely registered, the distances are 0 and the likelihood values are at their highest values, which is \( p(< x_j, x'i >) = 1 \). The choice of normal distribution in this model is a natural one assuming that the noise is normally distributed, such that the computed distances are concentrated around 0 in the case of precise registration. For such a small area, we did not consider the non-perpendicularity between the datum and the XY projection to be major; however, for larger areas this would be true and corrections applied.

The likelihood defined in Equation 4 takes \( x'i \) as its parameter which is computed using the collinearity Equations 1 and 2. Considering that the camera parameters in the collinearity equations contain \( p = [\omega, \psi, \kappa, Xc, Yc, Zc, f, x0, y0] \), the likelihood also takes \( p \) as its parameter and can be written as:

\[
p(< x_j, x'i >) = p(x'(p) | D) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{D(x'(p))^2}{2\sigma^2} \right)
\]

Applying Bayes’ rule to the conditional probability stated in this equation we have:

\[
p(x'(p) | D) = \frac{p(D | x'(p))}{p(D)}
\]

In Equation 6, the probability of generating a distance transform has a uniform distribution and the probability of projection of any ground data is equally likely, hence is constant, \( C \), such that \( p(x'(p) | D) = Cp(D | x'(p)) \). The posterior probability of distance transform \( D \) given projected ground data is computed based on the image data using Equations 4 and 5 and is computed from:

\[
p(x'(p) | D) = \frac{C}{\sigma \sqrt{2\pi}} \exp \left( -\frac{D(x'(p))^2}{2\sigma^2} \right)
\]

Figure 6. (a) Distance transform, \( D \), computed from a closed edge edge segment superimposed as white dotted line. Larger distances have lighter shade. (b) Euclidean distances computed from edge pixels that have a binary value of 1 in a neighborhood. The left matrix shows the edges with a value of 1. The right matrix shows the distance values from each edge pixel, a value of 0 is the edge.
We should note that the likelihood given in Equation 7 is not limited to normal distribution and can be replaced with other continuous distributions to model systematic camera errors or observation errors with known behavior.

The projection of edges from the lidar point cloud or the sampled GIS data will result in multiple points \( \mathbf{x} = (x'_1, x'_2, \ldots, x'_N) \) where \( N \) is the number of projected ground points. We assume that the ground data is observed independently, just like moving the leveling instrument during a turn at the end of the level route to create independent measurements. The probability given in Equation 7 can be extended to all points by:

\[
p(\mathbf{x}(\mathbf{p}) | D) = \prod_{i=1}^{N} p(x'_i(\mathbf{p}) | D) = \prod_{i=1}^{N} \frac{C_{2\sigma}}{\sigma \sqrt{2\pi}} \exp\left(-\frac{D(x'_i(\mathbf{p}))^2}{2\sigma^2}\right)
\]

Substituting Equation 7 into Equation 8, the likelihood of observing a set of projections for a given distance transform becomes:

\[
p(\mathbf{x}(\mathbf{p}) | D) = \prod_{i=1}^{N} \frac{C_{2\sigma}}{\sigma \sqrt{2\pi}} \exp\left(-\frac{D(x'_i(\mathbf{p}))^2}{2\sigma^2}\right) = \hat{C} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{N} D(x'_i(\mathbf{p}))^2\right)
\]

where \( \hat{C} = \left(\frac{C_{2\sigma}}{\sigma \sqrt{2\pi}}\right)^N \) is a constant. In this paper, our goal is to estimate the camera parameter vector \( \mathbf{p} \), such that, when \( \mathbf{p} \) is precise, the likelihood in Equation 10 is maximum. Hence, we define our cost function using the likelihood term in Equation 10. Considering that \( C \) and \( \hat{C} \) are constants, maximizing this likelihood term is equivalent to maximizing the posterior probability given on the right side of the equal sign in Equation 10.

Equivalently, the maximum a posteriori estimate defined above can be converted to a minimization problem by taking the negative logarithm of both parts of Equation 10. A benefit of this minimization problem is its numerical stability due to summation instead of multiplication of small probability values:

\[
l(\mathbf{p}) = -\log[p(\mathbf{x}(\mathbf{p}) | D)] = -\log \hat{C} - \frac{1}{2\sigma^2} \sum_{i=1}^{N} D(x'_i(\mathbf{p}))^2
\]

where \( l(\cdot) \) is the posterior term. Using this term, the solution to the camera parameter is given as:

\[
\hat{\mathbf{p}} = \arg \max_{\mathbf{p}} p(\mathbf{x}(\mathbf{p}) | D) = \arg \min_{\mathbf{p}} l(\mathbf{p}) = \arg \min_{\mathbf{p}} l(\mathbf{p}) = \arg \min_{\mathbf{p}} l(\mathbf{p})
\]

In this formulation, the estimate \( \hat{\mathbf{p}} \) can be computed following traditional minimization by setting the first derivative of the function to 0. This maximum a posteriori estimate problem resembles that of the least squares estimate problem in the case when there are no outliers (non-existent ground data), a constant variance (\( \sigma^2 \)) and independent observations with normally distributed errors (Myung, 2003; Olson, 2002). We should note that the scheme follows the same rules defined above if another model is selected. Rewriting Equation (14) in terms of the minimization problem results in:

\[
e = \sum_{i=1}^{N} D(x'_i(\mathbf{p}))^2
\]

and corresponds to the model given by:

\[
\mathbf{0}_{N \times 1} = \mathbf{d}(\mathbf{p}) + e
\]

where \( e \) is the error vector and the \( \mathbf{d} \) vector is given by:

\[
\mathbf{d} = |d_i| = D(x'_i(\mathbf{p})), \quad i \in \{1, \ldots, N\}
\]

The solution to Equation 16 can be computed by first linearizing Equation 16 and then estimating the parameters iteratively starting from an initial estimate, \( \mathbf{p}_0 \):

\[
0 = \mathbf{d}(\mathbf{p}_0) + J(\mathbf{d}(\mathbf{p})) \mathbf{p} \Delta \mathbf{p} + e
\]

where \( J(\mathbf{d}(\mathbf{p})) \) is the Jacobian of the vector-valued function \( \mathbf{d} \) and is a \( N \times 9 \) matrix in our problem. The projected ground data using the collinearity equations, \( x(\mathbf{p}) = (x'_i, y'_i) \) when introduced to \( J \), each row will have the following construct:

\[
J(d_i) = \begin{bmatrix} \frac{\partial D}{\partial x_{x'=x_i}} & \frac{\partial D}{\partial y_{y'=y_i}} \end{bmatrix}_{x'=x_i, y'=y_i}
\]

Therefore, the solution of \( \mathbf{p} \) is estimated using the generic least squares estimate as:

\[
\Delta \mathbf{p} = -(J^T) J^{-1} \mathbf{d}(\mathbf{p}_0)
\]

For a more stable solution with better convergence, the generic least squares solution can be altered with the Levenberg-Marquardt dampener:

\[
\Delta \mathbf{p} = -(J^T) (J J^T + \lambda \text{diag}(J^T J))^{-1} J^T \mathbf{d}(\mathbf{p}_0)
\]

where \( P \) is the weight matrix with entries computed from the distance, \( \lambda \) is the damping factor value used during minimization, and \( \text{diag}(\cdot) \) refers to the diagonal of the corresponding matrix. A large \( \lambda \) value (\%10) provides a stable, but slow convergence; while a lower \( \lambda \) allows for faster convergence, but may be unstable depending on the initial parameter values. When \( \lambda = 0 \) the solution reduces to least squares solution. The solution provided in Equation 19 allows the use of any type of data, not just image data, but the discrete points of the lidar data, or the points extracted from the polygon/polygon sets of GIS data. There is no need to find smooth patches or normals or parametric models for linear data. For estimating the camera parameters using Equation 20, we take a coarse-to-fine estimation strategy. This approach is taken due to change in the level of detail one can observe during edge detection. Particularly, at the coarse level, use closed edge segments extracted from the image as discussed in Section A. These coarser edges generally belong to buildings, as opposed to finer edges, which may represent cars and vegetation. The application of Equation 20 using generated from the coarse edges provides an intermediate solution of the camera parameters that are later used at the finer refinement step. At the fine level, we introduce more edges, including shorter ones to iteratively adjust the intermediate solution and provide precise camera parameters. The refinement achieved using the coarse-to-fine strategy is illustrated in Figure 7 by showing the projections.
of the ground GIS data on the image edges. It should be noted that if shorter more detailed edges are included in the initial adjustment, the coarse step, the solution will be biased due to incorrect initial matches between these edges and the projected ground data; hence the parameter estimation would result in an incorrect local solution.

**Experimental Results**

In order to show the performance of the proposed approach, we selected three experiment sites: one in California, San Bernardino (see Figure 1), one at The Ohio State University Campus (see Figure 8) and one at the Purdue University Campus (see Figure 10) for a high-oblique image. The GPS/IMU solutions for the camera orientations as well as the initial camera calibration parameters are provided with the images. These parameters are used as approximate initial solutions and are shown to improve when fused with the ground data in the form of lidar point cloud or discretized GIS vector data. The accompanying lidar and GIS data for the California site is provided from the San Andreas fault mapping project (Toth et al., 2006) and from San Bernardino county data and personal measurements on site, while The Ohio State site data is obtained from Ohio Department of Transportation, the Ohio Department of Natural Resources, and the Franklin County Auditor. All data is initially subjected to the processing steps previously sketched in an earlier Section. The Purdue data was taken by digital camera in the Mackey Arena, and initial calibration and orientation parameters were measured for the camera. Note that both coarse and fine level edges extracted from the image data is passed through the distance transform following the discussion in the previous Section. We realize that not all GIS data is of high quality data, but many counties/local governments use GIS on a daily basis for high quality solutions: Franklin County, Ohio or Hamilton County/Cincinnati, Ohio based CAGIS are created from survey/photogrammetric methods and are updated on a regular basis.

The camera parameters estimated from our approach are both quantitatively and qualitatively compared against the baseline traditional approach, where an operator selects a set of conjugate points between the image data and the ground data. As shown in Figure 9a and 9b, the set of points in the image space and their correspondences with the ground data is uniformly selected to reduce estimation bias. In our experiments, absolute camera parameters are adjusted using an <image,lidar> pair or an <image,GIS> pair. For either of these pairs, in the coarse to fine adjustment steps, we use a Levenberg damper > 1 and perform each level of refinement using a maximum 15 iterations. When the damper is set to 0, we observed convergence problems and the estimated parameters had higher standard deviations. For higher damper values, the camera parameters converge but at a slower rate.

For quantitative comparison, we provide two sets of reviews. The first set introduces the estimated parameters and the resulting standard deviations; while the second review measures quality of surface generation from the stereo-compilation using two images against the ground truth lidar data. We tabulate the initial and estimated camera parameters for the California study site in Tables 1, 2, and 3. The tables include estimated results for the manual adjustment, and the <image,lidar> and <image,GIS> pairs. The variance of unit weight \( \sigma_u \) parameter for manual, lidar-based and GIS-based adjustments respectively computed as 0.05406, 0.5967, and 0.9520.

The intrinsic camera parameters, as can be observed, do not change too much due to good initial estimates. The same observation, however, is not true for the extrinsic camera parameters. Specifically, there is on the average 50 cm change both vertically and horizontally. With respect to the standard deviation values, it can be observed that the manual

![Figure 7. Coarse to fine parameter estimation step by minimizing Equation 20 applied to GIS ground data. Local area is zoomed in to show the improvement. Projection using (a) initial camera parameters, (b) after coarse-refinement step, and (c) after fine-refined adjustment step.](image)
adjustment and the GIS based adjustment have similar orders of magnitude, while the lidar based adjustment has slightly higher standard deviation.

The second quantitative comparison is facilitated by first estimating the camera parameters for each image independently from the \(<\text{image, lidar}>\) pair and then using the estimated parameters to perform the stereo compilation between manually selected 32 corresponding points. The distances \(d_i\) between the recovered 3D points, \(\mathbf{x}_{\text{interact}}\) and the corresponding lidar points, \(\mathbf{x}_{\text{LIDAR}}\), serve as the comparison criteria. In Table 4, we tabulate the mean, \(\mu_d\), and the standard deviation, \(\sigma_d\), for the distances computed using three different camera parameter estimation methods: (a) manual, (b) \(<\text{image, lidar}>\) pair, and (c) \(<\text{image, GIS}>\) pair. Table 5 tabulates the quantitative comparison results for the Ohio State University study site for all three methods. The variance of unit weight \(\sigma_0\) parameter for manual, lidar-based and GIS-based adjustments respectively computed as 0.0312, 0.3700, and 0.2475. Similar to the California results, the results from the proposed approach is better than the manual approach. We should note that the intrinsic camera parameters comparably change more than results for the California study site. Overall, we should note that, while better than manual approach, the OSU site results are not as good as the California site. This observation is primarily due to the extreme scale difference between the imagery used for both sites, as well as the density of structures in the environment. For instance, the buildings outside the OSU campus area are only a few feet apart making even manual selection of control points difficult. This observation extends to the lidar and GIS data where all buildings generate a repetitive pattern and distinguishing them from one another is very hard. This observation is manifested in the 3D recovery results tabulated in Table 6. While not better than the manual approach as it is in the results shown for the California site, the estimated 3D, when GIS data is used, is still comparable to the manual results and improves the results computed using the initial camera parameters. The result estimated using the lidar data, however, is not as good due to fusing sparse lidar data with very low scale image whose edges do not align well after the extraction process.

Table 4. Mean distances computed in meters for \(x\), \(y\), and \(z\) coordinates between lidar points and recovered 3D points for the California study site; the recovered points are estimated by intersecting the rays from corresponding points between two images whose camera parameters are estimated using the proposed method.

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<td>Y</td>
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<td>(\sigma_d)</td>
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<td>0.322</td>
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Figure 8. (a) Image used in our experiments for the OSU study site. The “Horse Shoe” shaped Ohio stadium is on the top left of the image; (b) OSU GIS data (cropped for better visualization) that corresponds to part of the image in part (a). The GIS data contains the buildings, street centerlines and curbs; note that the stadium on the top left; (c) Local lidar elevation image for the same study site with stadium in top middle. Only local data is shown for better visualization.

Figure 9. An example subset of point correspondences used to facilitate a comparison of our automated approach against traditional manual approach: (a) Subset of selected points marked with \(\times\) (white cross) in the image space, and (b) Corresponding 3D points marked with \(\bullet\) (black dot) superimposed on the lidar data.
High Oblique Experimental Results

The high-oblique results also show success with the proposed method. The image was taken inside Mackey Arena at Purdue University (Figure 10). Ground control features (10 cm radius circles) were laid on the surface of the basketball floor and the centroids measured of these features. These circular features are embedded in nine separate targets on the floor. Each target had three ground control features on it (see Figure 11). Each control feature was labeled \( <a, b> \) or \( <c> \). Initial orientation and calibration values were measured and calculated for the camera. While the setup is different than the previous methodologies (there is no GIS or lidar data), but the ground control features do provide a similar setup: the control features are not dimensionless points, as in an ideal GCP in photogrammetry, but they are 10 cm radius circle features. These features can be then presumed to be the GIS features, since with edge detection the outlines can be easily obtained for the images; and since the centroid and radius is known of these features the outline of the ground control features can also be computed, giving an outline similar to GIS features. The elevations of the circles are constant, as the basketball arena floor is considered level. Thus the XYZ coordinates of the outlines are known in object space and the x, y coordinates of the outlines are known in image space.

Using the initial orientation and calibration data, the results converged to a final camera orientation and calibration value. Even though the image was taken at a high-oblique angle, this did not make a difference in the performance of the method. The results tabulated in Table 7. In both the low and high-oblique cases, we have found that the dampening factor \( \lambda = 1 \) seems to work the best. A lower value may not always converge; a higher value takes too long.
Table 7. Solutions for (a) the principal point offset and focal length all in millimeters; (b) perspective center coordinates in meters; and (c) camera orientation in degrees; the results are generated for the Purdue University study site.

(a) Method | $f$ | $x_0$ | $y_0$ | $\sigma_x$ | $\sigma_y$ | $\sigma_{xy}$
---|---|---|---|---|---|---
Manual | 4.9287 | -0.1311 | 0.0651 | 0.0033 | 0.0054 | 0.001

(b) Method | $X_0$ | $Y_0$ | $Z_0$ | $\sigma_{x_0}$ | $\sigma_{y_0}$ | $\sigma_{z_0}$
---|---|---|---|---|---|---

(c) Method | $\omega$ | $\varphi$ | $\kappa$ | $\sigma_\omega$ | $\sigma_\varphi$ | $\sigma_\kappa$
---|---|---|---|---|---|---
Manual | 78.6844 | -27.8288 | -1.4672 | 0.0698 | 0.0569 | 0.0539

Conclusions

This research introduces a novel approach to estimate camera parameters by registering images with GIS or lidar data. Compared to many other methods in the literature, the proposed approach does not require conjugate points across data-sets; hence is not affected by projective changes in appearances. Instead, our approach uses freeform edges and lines, which are hypothesized to have correspondence across modalities. The approach is based on the use of the collinearity model, subjected to stochastic modeling of projections of 3D points onto the distance transform for computing hypothesized residuals. The sum of these residuals defines a cost function, minimized by exploiting the Levenberg-Marquardt dampener. The proposed approach is general and can be applied to pair to estimate relative orientation or it can be applied to register other sensory data, such as synthetic aperture radar (SAR), to images. In our experimental setup, we have excluded lens distortion parameters during the estimation; however, we should note that these parameters can also be included in the process without added complexity. In our experimental setup, we studied two different sites with three different information modalities, which share the same reference frame: electro-optical image, lidar, and GIS. Our results on these study sites show that the registration of images with lidar and GIS can be automated and the information automatically extracted from each data is compatible with each other, such that it can be used for precise registration. Our comparisons of both results against the traditional manual approach, where an operator is required to select conjugate points across the modalities, show promising results and have comparable quantitative performance. Image based photogrammetry is not “dead”, far from it. While lidar has gained enormous momentum over the last few years, it has several drawbacks when compared to image-based solutions. For an excellent comparison of lidar and image based solutions, we refer the reader to the article by Leberl et al. (2010). Images provide useful data not available in lidar data. Future work will include bundle adjustment self-calibration comparison to proposed method. Application areas could be at construction sites, which do not have distinctive point features but have line features.

Acknowledgments

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References

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